A PDE APPROACH TO FRACTIONAL DIFUSSION: A PRIORI AND A POSTERIORI ERROR ANALYSES

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We study PDE solution techniques for problems involving fractional powers of symmetric coercive elliptic operators in a bounded domain with Dirichlet boundary conditions. These operators can be realized as the Dirichlet to Neumann map of a degenerate/singular elliptic problem posed on a semi-infinite cylinder, which we analyze in the framework of weighted Sobolev spaces. Motivated by the rapid decay of the solution of this problem, we propose a truncation that is suitable for numerical approximation. We discretize this truncation using first order tensor product finite elements. We derive a priori error estimates in weighted Sobolev spaces, which exhibit optimal regularity but suboptimal order for quasi-uniform meshes and quasi-optimal order-regularity for anisotropic meshes. As a first step towards adaptivity, we also present a computable a posteriori error estimator which relies on the solution of small discrete problems on stars. The estimator exhibits built-in flux equilibration and is equivalent to the energy error up to data oscillation. We design a simple adaptive strategy, which reduces error and data oscillation, and present numerical experiments to illustrate the a priori and a posteriori error estimates.

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