THE RIESZ POTENTIAL AS A MULTILINEAR OPERATOR INTO BMO_β SPACES

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Abstract. For $\alpha > 0$, we consider the standard Riesz type fractional integral defined by convolution with the kernel $K_\alpha(y_1, \ldots, y_k) = (|y_1| + \cdots + |y_k|)^{\alpha - kn}$, on the tensor product of the entries and restrict it to the diagonal of $(\mathbb{R}^n)^k$, (n and $k \geq 2$ fixed positive integers). More precisely, for $x \in \mathbb{R}^n$,

$$I_{\alpha,k}\vec{f}(x) = \int_{\vec{y} \in (\mathbb{R}^n)^k} \frac{f_1(y_1) \ldots f_k(y_k)}{(|x - y_1| + |x - y_2| + \cdots + |x - y_k|)^{(kn-\alpha)}} d\vec{y} \qquad (0.1)$$

for a k-dimensional vector field $\vec{f} = (f_1, f_2, \ldots, f_k) \in L^{p_1} \times \cdots \times L^{p_k}$ and $\vec{y} = (y_1, \ldots, y_k) \in (\mathbb{R}^n)^k$. Kenig and Stein showed in [KS] that, when $\sum_{i=1}^k \frac{1}{p_i} - \frac{\alpha}{n} > 0$, the target space of this operator is L^q with $\frac{1}{q} = \sum_{i=1}^k \frac{1}{p_i} - \frac{\alpha}{n}$.

We consider the opposite situation, when $\sum_{i=1}^k \frac{1}{p_i} - \frac{\alpha}{n}$ vanishes or is negative, and show that the target space is the space BMO_β defined through mean oscillations, with $\beta = \alpha - \sum_{i=1}^k \frac{n}{p_i}$, recovering from the linear case, the result proved in [HSV]. Since BMO_β is a space of classes modulo polynomial of order $[\beta]$ we redefine the fractional integral in order to guarantee the convergence of the integral when $\sum_{i=1}^k \frac{1}{p_i}$ is small.

References

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