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Existence, uniqueness and stability of minimizers of generalized Tikhonov-Phillips functionals

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Abstract. The Tikhonov-Phillips method is widely used for regularizing ill-posed inverse problems mainly due to the simplicity of its formulation as an optimization problem. The use of different penalizers in the functionals associated to the corresponding optimization problems has originated a variety other methods which can be considered as "variants" of the traditional Tikhonov-Phillips method of order zero. Such is the case for instance of the Tikhonov-Phillips method of order one, the total variation regularization method, etc. In this article we find sufficient conditions on the penalizers in generalized Tikhonov-Phillips functionals which guarantee existence, uniqueness and stability of the minimizers. The particular cases in which the penalizers are given by the bounded variation norm, by powers of seminorms and by linear combinations of powers of seminorms associated to closed operators, are studied. Several examples are presented and a few results on image restoration are shown.

Keywords: Inverse problem, Ill-Posed, Regularization, Tikhonov-Phillips.

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