UNIFORM SPACES AND THE NEWTONIAN STRUCTURE OF (BIG)DATA AFFINITY KERNELS

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ABSTRACT. Let X be a (data) set. Let $K(x, y) > 0$ be a measure of the affinity between the data points x and y. We prove that K has the structure of a Newtonian potential $K(x, y) = \varphi(d(x, y))$ with φ decreasing and d a quasi-metric on X under two mild conditions on K. The first is that the affinity of each x to itself is infinite and that for $x \neq y$ the affinity is positive and finite. The second is a quantitative transitivity; if the affinity between x and y is larger than $\lambda > 0$ and the affinity of y and z is also larger than λ, then the affinity between x and z is larger than $\nu(\lambda)$. The function ν is concave, increasing, continuous from $\mathbb{R}^+$ onto $\mathbb{R}^+$ with $\nu(\lambda) < \lambda$ for every $\lambda > 0$.