BOUNDEDNESS OF OPERATORS RELATED TO A DEGENERATE SCHRÖDINGER SEMIGROUP

E. HARBOURE, O. SALINAS, AND B. VIVIANI

ABSTRACT. In this work we search for boundedness results for operators related to the semigroup generated by the degenerate Schrödinger operator $\mathcal{L}u = -\frac{1}{\omega}\operatorname{div} A \cdot \nabla u + V u$, where ω is a weight, A is a matrix depending on x and satisfying $\lambda\,\omega(x)|\xi|^2 \le A(x)\cdot\xi\cdot\xi \le \Lambda\,\omega(x)|\xi|^2$ for some positive constants λ, Λ and all x, ξ in $\mathbb{R}^d$, assuming further suitable properties on the weight ω and on the non-negative potential V. In particular, we analyze the behaviour of T^*, the maximal semigroup operator, $\mathcal{L}^{-\alpha/2}$, the negative powers of $\mathcal{L}$, and the mixed operators $\mathcal{L}^{-\alpha/2}V^{\sigma/2}$ with $0 < \sigma \le \alpha$ on appropriate functions spaces measuring size and regularity. As in the non degenerate case, i.e. $\omega \equiv 1$, we achieve these results by first studying the case $V = 0$, obtaining also some boundedness properties in this context that we believe are new.